[6] Lomax, H., "An Operational Unification of Finite Difference Methods for the Numerical Integration of Ordinary Differential Equations," TR-R-262, 1967, NASA.

[7] Krishnamurthy, L. and Williams, F. A., "Kinetics and Regression," *SIAM Journal of Applied Mathematics*, Vol. 20, 1971, pp. 590–611.

Supersonic Laminar Base Pressure, Heat-Transfer, and Upstream Influence Correlations for Small Steps

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Introduction

MOST studies of supersonic boundary-layer flow past a rearward-facing step have dealt primarily with the Chapman-Korst limit where the step height h is large compared to the incoming boundary-layer thickness δ and the resulting expansion around the corner is essentially a rotational inviscid flow problem. Far less has been done for high Reynolds number flows in the opposite limit of a small step such that $h/\delta \ll 1$. However, this latter situation is of considerable practical interest; for example, in connection with structural skin joint faults on large high-speed vehicles such as the Space Shuttle where the heat-transfer and pressure disturbances around such small steps or gaps is of concern.

This Note describes some results of the application of a unified theory of small disturbances in nonuniform high-speed boundary layers[1] to this small-step problem for the case of supersonic laminar nonadiabatic flow. Comparisons with experimental data will be given which show good agreement in a variety of physical features, including useful new base pressure and heat-transfer correlation relations.

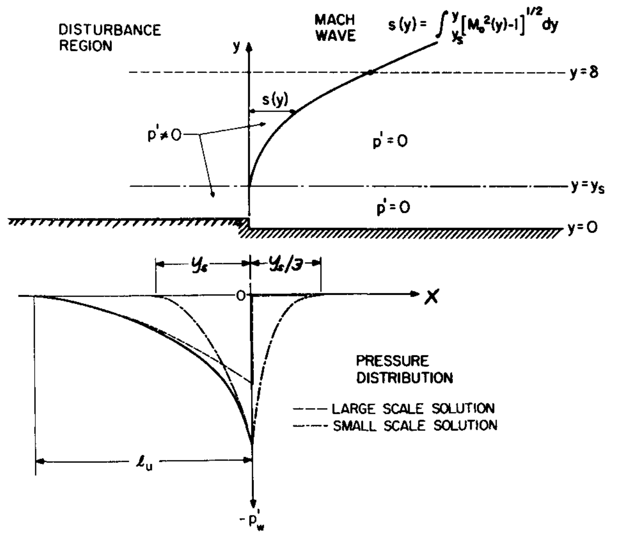

Fig. 1 Large scale features of pressure disturbance field (schematic).

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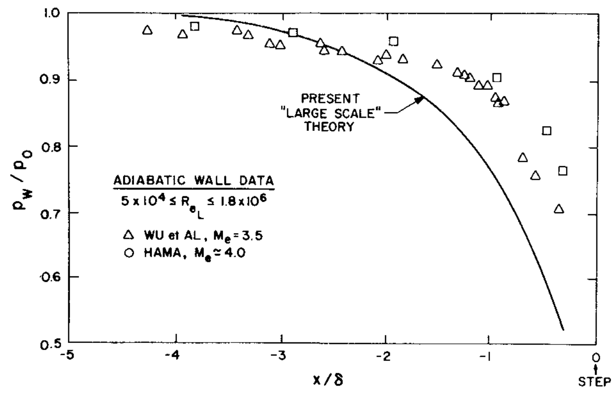

Fig. 2 Wall pressure distribution upstream of step.

Theory and Experiment

Using a small perturbation approach, an analytical theory was developed that predicts the steady-state pressure, temperature, and heat-transfer disturbance fields including suction or blowing through the step.[2] Particular emphasis was placed on the important effects of the highly nonuniform flow across the boundary layer, lateral pressure gradients, and the upstream influence associated with viscous-inviscid interaction. Approximate analytical expressions for the wall pressure, skin-friction, and heat-transfer perturbations and upstream influence distance were derived by a Fourier transformation approach.

The leading approximation for the large-scale pressure disturbance field along the wall is found[2] to vanish downstream of the step $x > 0$ (Fig. 1), while upstream behaving like

$$p_w'/\gamma P_o \approx (-h/l_u)[M_o^2(y^*)/(M_{o_e}^2 - 1)^{1/2}]e^{x/l_u} \quad (1)$$

where $M_o(y^*)$ is the boundary-layer profile Mach number at the edge $y = y^*$ of the Lighthill viscous disturbance sublayer and l_u is the upstream distance given by

$$u_e l_u/\nu_e \approx 2.9(T_w/T_e)^{5/4}(Cf_o)^{-5/4}(M_{e_o}^2 - 1)^{-3/8}$$

with

$$y^*/l_u \approx 0.49 C_{f_o}^{1/2}(M_{e_o}^2 - 1)^{1/4}$$

and C_{f_o} is the undisturbed flow skin-friction coefficient. A comparison of this prediction for laminar flow with some experimental upstream wall pressure distributions[3] is shown in Fig. 2. The boundary-layer anticipation of the corner expansion is seen to be reasonably well described by the present linearized theory with an over-all upstream influence distance that is relatively insensitive to the step height.

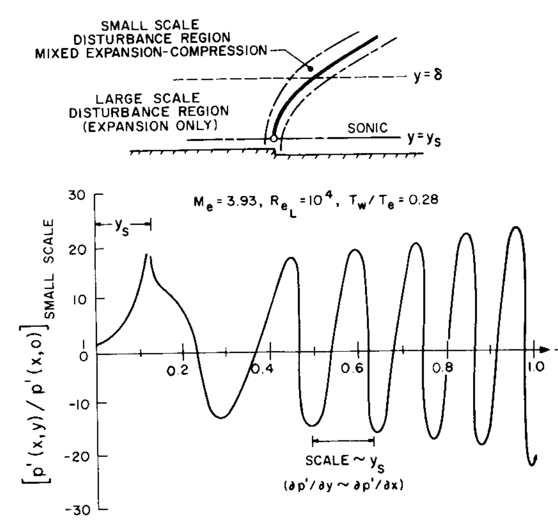

Fig. 3 Small-scale perturbation structure.

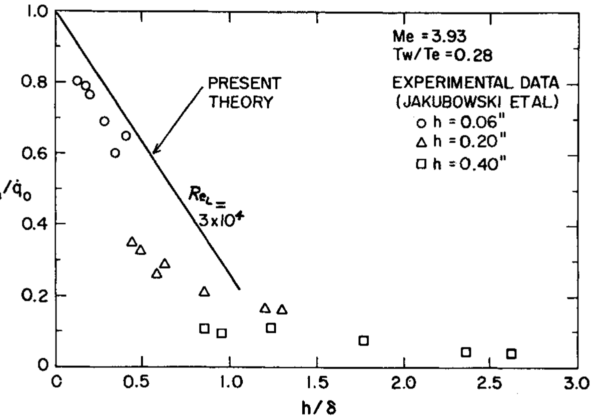

Fig. 5 Base heat transfer vs step height/boundary-layer thickness ratio.

The corresponding small-scale wall pressure solution has the form

$$p_w'/\gamma P_o \approx -(4h/\pi y_s)M_o^2(y^*)e^{kx/y_s} \tag{2}$$

in terms of the boundary-layer sonic height y_s where $k = 1$ and -3 for $x < 0$ and $x > 0$, respectively. This variation is continuous past the step (albeit with a discontinuous axial *gradient*) and acts to locally smooth out the large scale pressure jump illustrated in Fig. 1. Moreover, whereas the lateral pressure gradient in the large scale is negligibly small, it is quite large and rapidly varying (in fact comparable to the local axial pressure gradient) near the corner, as shown in Fig. 3.

Assuming laminar boundary-layer flow, the following useful closed-form approximation for the base pressure $p_B = p_w'(x \to 0^+)$ can be derived[2] from Eq. (2):

$$\frac{p_B}{p_o} \approx 1 - C_1 \frac{M_e^3 Re_L^{1/4}}{(M_e^2-1)^{1/4}} \frac{h}{L} = 1 - C_1 \Lambda \tag{3}$$

where the constant C_1 depends on T_w/T_e. Equation (3) indicates that the quantity Λ is a basic correlating parameter for the base pressure. This is verified in Fig. 4, where several sets of experimental data[4,5] are shown to correlate in terms of a single curve function of Λ over a moderately wide range of laminar flow conditions.

Solution of the energy equation in the heat-conducting disturbance sublayer near the wall gives the corresponding heat transfer behavior.[2] In the small-scale approximation near the step, this yields the following approximate laminar base heat-transfer expression:

$$\frac{q_B}{q_{w_o}} = 1 - C_2 \frac{M_e^2 Re_L^{5/8}}{(M_e^2-1)^{1/2}} \frac{h}{L} \tag{4}$$

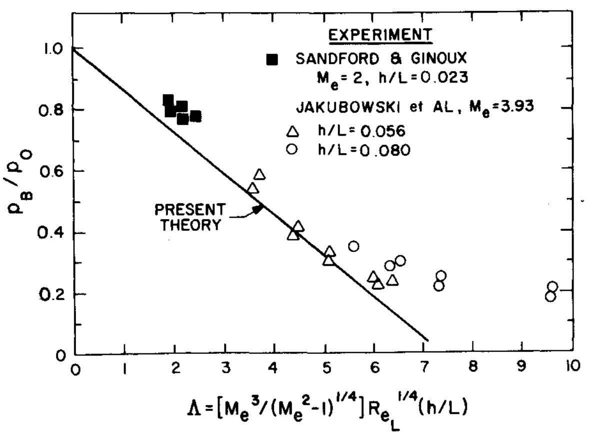

Fig. 4 Base pressure correlation for laminar supersonic flow.

which, as expected, exhibits a much stronger dependence on the Reynolds number than base pressure. This theoretical result is plotted vs h/δ in Fig. 5 along with some recent experimental measurements.[4] Although the predicted linear behavior at small h/δ is confirmed, the theory overestimates the heat transfer noticeably more than it does base pressure. This is to be expected since the presence of a recirculation zone downstream of the step (which is not accounted for in the theory) significantly reduces base heating for the cited step heights.

References

[1] Inger, G. R., "Three-Dimensional Heat Transfer and Ablation Disturbances in High Speed Flows," *AIAA Journal*, Vol. 10, No. 12, Dec. 1972, pp. 1641–1646.

[2] Inger, G. R., "Theory of Supersonic Laminar Non-Adiabatic Boundary Layer Flow Past Small Rearward-Facing Steps Including Suction," Rept. VPI-E-72-17, Aug. 1972, Virginia Polytechnic Institute and State University, Blacksburg, Va.

[3] Wu, J. M., Su, M., and Moulden, T. H., "On the Near Flow Field Generated by the Supersonic Flow over Rearward-Facing Steps," ARL-71-0243, Nov. 1971, Aerospace Research Labs., Wright-Patterson Air Force Base, Ohio.

[4] Jakubowski, A. K., Brown, R. D., Kirchner, R. D., and Lewis, C. H., "Experimental Investigation of Heat Transfer and Pressure Distributions in Laminar Separated Flows Downstream of a Rearward-Facing Step," Aerospace Engineering Report, July 1973, Virginia Polytechnic Institute and State University, Blacksburg, Va.

[5] Sandford, J. and Ginoux, J. J., "Laminar, Transitional, and Turbulent Heat Transfer Behind a Backward-Facing Step in Supersonic Flow," TN 38, Oct. 1968, von Kàrmàn Institute, Brussels, Belgium.

Generalized Sturm-Liouville Procedure for Composite Domain Anisotropic Transient Conduction Problems

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Introduction

THE transient and steady-state temperature distribution in composite configurations consisting of several distinct thermally anisotropic subdomains have numerous applications to heat-transfer problems in re-entry vehicles, air frames, nuclear reactors, and the like. Apart from the purely numerical approaches such as the finite element[1–3] and difference procedures,[4] several purely analytical techniques[4–8] are also available. A comprehensive survey of the above noted analytical procedures has been reported by Ozisik.[4] With few exceptions,[9,10] most applications of analytical techniques have been limited to isotropic 1-D laminated domain problems. This is partly due to the added analytical complexity caused by anisotropy as well as the general lack of attention given to such thermal material properties. Due to the increased usage of inherently thermally anisotropic materials in a variety of applications, this situation is changing.

In this context, the purpose of the present Note will be to extend the relatively simple and straightforward Vodicka-Tittle[4–6] orthogonal expansion technique to 3-D configurations consisting of finitely many distinct fully anisotropic subdomains. As will be seen in the development, the procedure developed herein is based on a 3-D piecewise weighted orthogonality

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